Revise :- Parallelogram law of forces.

Cor 3 :- Direction of resultant of two concurrent forces is inclined more towards the greater force.

Proof :- Let $P > Q$

Then, $P + Q\cos\alpha > Q + Q\cos\alpha$ (adding $Q\cos\alpha$ both sides).

$$\Rightarrow \tan\theta = \frac{Q\sin\alpha}{P+Q\cos\alpha} < \frac{Q\sin\alpha}{Q+Q\cos\alpha} = \frac{\sin\alpha}{1+\cos\alpha}$$

$$\Rightarrow \tan\theta < \frac{\sin\alpha}{1+\cos\alpha} = \frac{2\sin\alpha/2\cos\alpha/2}{2\cos^2\alpha/2} = \tan\alpha/2$$

$$\Rightarrow \theta < \frac{\alpha}{2}$$

Hence, the direction of resultant of two concurrent forces is inclined more towards the greater force.

Cor 4 :- The maximum magnitude of the resultant of concurrent forces of magnitude P and Q is $P+Q$.

Proof :- We have,

$$R = \sqrt{P^2+Q^2+2PQ\cos\alpha}.$$

Now, R is maximum if $\cos\alpha = 1$

i.e. $\alpha = 0^\circ$

$$\therefore R_{max} = \sqrt{P^2+Q^2+2PQ} = \sqrt{(P+Q)^2} = P+Q$$

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$\Rightarrow R_{max} = P+Q$.

Hence, maximum magnitude of resultant is $P+Q$, and it acts along the same direction of given forces.

cor. 5 :- The minimum magnitude of the resultant of concurrent forces P and Q is $|P-Q|$.

Proof :- We have,

$$R = \sqrt{P^2+Q^2+2PQ\cos\alpha}$$

Now, R is minimum if $\cos\alpha = -1$

$$\Rightarrow \alpha = \pi$$

$$\therefore R_{min} = \sqrt{P^2+Q^2-2PQ} = \sqrt{(P-Q)^2} = |P-Q|$$

$$\Rightarrow R_{min} = |P-Q|$$

Hence, minimum magnitude of the resultant of concurrent forces is $|P-Q|$ and it acts in the direction of the greater force.

cor. 6 :- Two non-zero forces acting on a particle are in equilibrium if they are of equal magnitude and opposite directions.

Proof :- If $R=0$, then

$$\left.\begin{aligned} P+Q\cos\alpha &= R\cos\theta = 0 \\ \text{and } Q\sin\alpha &= R\sin\theta = 0 \end{aligned}\right\} \quad \text{——— (1)}$$

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$\Rightarrow P+Q\cos\alpha=0$ and $Q\sin\alpha=0$

$\because Q\sin\alpha=0$

$\Rightarrow \sin\alpha=0 \quad [\because Q\neq 0]$

$\Rightarrow \alpha=0$ or π.

when $\alpha=0$

$\Rightarrow P+Q\cos\alpha=0$

$\Rightarrow P-Q=0$

$\Rightarrow P=Q$

Hence, two non-zero forces acting on a particle are in equilibrium if they are of equal magnitude and opposite directions.

Remark:- If two forces $\vec{P}$ and $\vec{Q}$ acts at angle α, and $\vec{R}$ is their resultant is of making an angle θ with $\vec{P}$, then

$$\sin\theta=\frac{Q\sin\alpha}{R} \quad \text{and} \quad \cos\theta=\frac{P+Q\cos\alpha}{R}.$$

Conclusions:-

1. It is noted that the magnitude of the resultant of two vectors is maximum, when the vectors act in the same direction and is minimum when they act in opposite directions.
2. It should be noted that while finding the resultant vector of two vectors by the parallelogram law of vector addition, the two vectors A and B should be either act towards the point or away from point.

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History :-

Stevinus (1548 - 1620) was the first to demonstrate that forces could be combined by representing them by arrows to some suitable scale, and then forming a parallelogram in which the diagonal represents the sum of two forces. In fact, all vectors must combine in this manner.

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